JAMES

Theory of the Transformer

Electrical Engineering

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THEORY OF THE TRANSFORMER

BY

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B. S. UNIVERSITY OF ILLINOIS, 1906

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Leonard Vaughan James.

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THEORY OF THE TRANSFORMER

INTRODUCTION

The predetermination of the operating characteristics of the transformer, from a knowledge of its constants and of the load conditions, is largely a study of simple physical relations. This is true in the transformer because of the actual constancy in value of the so-called "constants", a condition due especially to the presence of a low permeability path for the main or exciting field and of high and constant permeability paths for the leakage fields, as well as to the fact that there is no shifting of these fields between paths of different permeabilities when the load changes. Because of these things, the physical relations when determined from simple short circuit and open circuit tests, may be considered as the general relations for any condition of loading. This makes possible, therefore, the use of comparatively simple mathematical equations and a correspondingly simple solution of the problem.

As will appear later, the most essential principle upon which the operation of the transformer is based is that of mutual induction, requiring differential equations in its general solution. Thus the machine will first be considered

from this standpoint. The solution will then be obtained graphically by the substitution of equivalent sine waves for the several more or less irregular waves representing the alternating voltages and currents. It will be seen that the vector diagram presents the graphical solution more clearly than the waves but that both waves and vectors have the disadvantage of being inaccurate. When the vectors are expressed as general numbers, however, and the problem solved by the algebra of the complex quantity, the clearness of the vector analysis is combined with the accuracy of exact analytical solution. The only limitation in this method is that equivalent sine waves must be substituted for the original waves. Whenever this is impossible, the problem can only be solved by resorting to the use of differential equations.

DEVELOPMENT OF DIFFERENTIAL EQUATIONS

Principles and Construction of the Transformer

A transformer is a machine in which the principle of mutual induction between stationary coils is utilized in transferring electrical energy from one circuit to another. In general, the number of turns is not the same in the two coils so that the energy will appear in the second coil at a different voltage from that in the first, this transformation being responsible for the name, transformer. Depending upon induction, this machine is used only in alternating current circuits and can be used only where the energy supplied is alternating or oscillatory in nature.

In mechanical construction, the transformer consists of two coils, a primary and a secondary, connected by an interlinking magnetic circuit. Due to the resistances of the electric circuits and to any hysteretic effects in the magnetic circuits, there will be a certain loss of energy, while the periodic storage and release of energy in the leakage magnetic fields will affect the phase displacement between the voltage and current.

A survey of the physical phenomena involved in this machine shows that, when an alternating voltage is impressed upon the primary coil a current flows in that coil, causing an alternating flux to be set up in the magnetic circuit.

This flux induces an electro-motive-force both in the primary and in the secondary winding, opposing that impressed in the primary and providing an electro-motive-force in the secondary which is available to send a current through a second circuit. Should this current be permitted to flow, it will react upon the magnetic field according to Lenz's Law in such a way as to decrease the same and finally to cause an increase in the current in the primary winding. The currents thus appearing in the primary and secondary coils are known by the names of the respective windings as primary and secondary currents. Likewise the primary voltage is the voltage impressed upon the terminals of the primary winding and the secondary voltage is that which appears at the terminals of the secondary.

Mathematical Relation, Referred to the Primary Coil

In developing the mathematics expressing the fundamental relations in the transformer, the primary coil will first be considered alone. As before stated, if a voltage be impressed upon this coil a current will flow through it. The relation between this voltage and current is expressed in the fundamental differential equation of the electric circuit,

$$e_1 = i_1 r_1 + L_1' \frac{di_1}{dt} \qquad (1)$$

where

e_1 = impressed voltage at any instant, t,

i_1 = current in primary at the same instant, t,

r_1 = resistance of primary coil,

L'_1 = total inductance of primary coil.

The inductance, L'_1, is here the total inductance of the primary coil and is a quantity depending upon the dimensions and material of the magnetic circuit. One of these dimensions is seen to be the entire cross-sectional area of the coil.

It is apparent that a part of the total flux due to the primary current will induce a voltage in the secondary coil so that, if this coil be closed through a resistance, a current i_2 will flow. The problem now becomes one of imperfect mutual induction, and

$$e_1 = i_1 r_1 + L'_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad (2)$$

where M is the coefficient of mutual induction.(See note, p.13.)

In (2), L'_1 is still the inductance due to the entire area of the primary coil. If the flux in any coil is separated into a number of parts, in order to consider the various paths or for any other reason, the sum of the voltages induced by each part must be the e. m. f. which would be induced by the total flux.

That is, if

$$\varphi = \varphi_a + \varphi_b + \varphi_c + \text{---}$$

or if *

$$\varphi = Ki = K_a i + K_b i + K_c i + \text{---}$$

*Paths in air having constant inductance for different densities are here intended. Were an iron path introduced this mathematics would not apply exactly as shown but the reasoning and final conclusions would be similar.

then

$$-n\frac{d\varphi}{dt} = -nk\frac{di}{dt} = -nK_a\frac{di}{dt} - nK_b\frac{di}{dt} - nK_c\frac{di}{dt} \quad ---$$

Where φ is flux, i is current, n is turns and K is a constant depending upon the dimensions of the coil and the part of the area under consideration. Then, by the definition of the coefficient of self-induction,

$$L = L_a + L_b + L_c + --- \qquad (3)$$

where the values of L are calculated for their respective parts of the area.

From (3) it appears that (1) may be written as follows,

$$e_1 = i_1 r_1 + L_1\frac{di_1}{dt} + L_{o_1}\frac{di_1}{dt} \qquad (4)$$

where L_{o_1} is the inductance factor for that part of the flux which threads both coils and which will here be termed the useful or exciting flux because it induces the secondary e. m. f., while L_1 is the inductance factor for the rest of the flux threading the primary coil, which, since it performs no useful function, is termed the leakage flux. This idea may also be expressed in (2), so that

$$e_1 = i_1 r_1 + L_1\frac{di_1}{dt} + L_{o_1}\frac{di_1}{dt} - M\frac{di_2}{dt}. \qquad (5)$$

"Induced Voltage" and Reactance Drop

In (5) it is seen that the total voltage consumed by induced e. m. f.'s is

$$L_1 \frac{di_1}{dt} + L_{o_1} \frac{di_1}{dt} - M \frac{di_2}{dt}.$$

Of this, that voltage consumed by the e. m. f. induced by the useful flux or what is commonly termed the "induced voltage" in the primary is

$$e_{i_1} = L_o \frac{di_1}{dt} - M \frac{di_2}{dt}. \qquad (6)$$

The remaining part, $L_1 \frac{di_1}{dt}$, is consumed by the e. m. f. induced by the leakage flux and is known as the reactance drop or leakage reactance drop.

By substitution from (6), (5) now becomes

$$e_1 = i_1 r_1 + L_1 \frac{di_1}{dt} + e_{i_1} \qquad (7)$$

This equation is very suggestive. If perfect mutual induction and zero resistance may be assumed, it becomes

$$e_1 = e_{i_1} = L_{o_1} \frac{di_1}{dt} - M \frac{di_2}{dt}$$

which shows that, in this case and for the same impressed voltage, the exciting flux will be the same whatever the value of the secondary resistance and current, the primary current changing to maintain the equilibrium. Under the natural or imperfect conditions it is seen that e_{i_1} will be less than e_1 by

$$i_1 r_1 + L_1 \frac{di_1}{dt}$$

or by the resistance and reactance drops in the primary.

By substituting for M, (5) may be re-written so as to show two very important relations. The mutual inductance is

$$M = \sqrt{L_{o_1} L_{o_2}} \tag{8}$$

where L_{o_2} is the inductance of the secondary coil for the dimensions of the exciting field. From this definition it follows that

$$L_{o_2} = \left[\frac{n_2}{n_1}\right]^2 L_{o_1} \tag{9}$$

and so by substitution

$$M = \frac{n_2}{n_1} L_{o_1} .$$

Substituting this value of M into (5) and collecting,

$$e_1 = i_1 r_1 + L_1 \frac{di_1}{dt} + L_{o_1} \left[\frac{di_1}{dt} - \frac{n_2}{n_1} \frac{di_2}{dt}\right] \tag{10}$$

Relative Magnitudes of Induced and Impressed Voltages

In (10) it first appears that, if the induced primary e. m. f. is to approach the impressed voltage, the resistance and reactance drops must be very small when compared with e_{i_1}, which requires that L_1 must be very small when compared with L_{o_1}. This relation is obtained by providing an iron core for the exciting flux and by winding the coils closely over this core. In many cases, the primary and secondary windings are made up of several coils each and these coils are "sandwiched" between each other so as to provide a still greater ratio of L_{o_1} to L_1. In some cases,

however, definite provision is made for a large value of L_1 as a protective feature against surges or short-circuits.

The Exciting Current

The second relation to be found in (10) is that, if there is to be an exciting flux to induce e_{i_1},

$$n_1 \, di_1 > n_2 \, di_2$$

or

$$n_1 \, i_1 > n_2 \, i_2 \, .$$

Interpreted, this means that the primary m. m. f. must be greater than the secondary m. m. f. by enough to produce the exciting flux. The additional primary current necessary is known as the exciting current and is designated by i_{oo}, where

$$i_{oo} = i_1 - \frac{n_2}{n_1} i_2 \, . \qquad (11)$$

For an air core this exciting current will have the same wave form as the flux it produces and will be in phase with that flux; for an iron core its wave form and phase relation are distorted when compared with those of the flux, due to the hysteresis and saturation effects in the iron. To be exact in the case of the iron core, then, the exciting current can not be assumed to vary directly either with the flux or with the induced e. m. f. Its wave shape will also be likely to change for different values of maximum flux density even though the flux waves have the same shape.

Since, however, the transformer is usually worked well below the knee of the saturation curve and since the maximum range of variation of the effective value of e_{i_1} is from e_1 on open circuit to half of e_1 on short circuit, when the exciting current is a small percent of the rated current, the approximation

$$i_{oo} = K e_{i_1} \tag{11a}$$

can be made without material error in the results. When this is done, the exciting current is usually resolved into two components, the core loss current in phase with the induced voltage being that required to supply the hysteresis and eddy current losses in the iron, and the magnetizing current in phase with the flux being that required to magnetize the core.

Final Equation for the Primary

If the relation of currents as expressed in (11) is substituted into (10) and the equation made to depend only upon the primary and exciting currents, the final expression is

$$e_1 = i_1 r_1 + L_1 \frac{di_1}{dt} + L_{o_1} \left[\frac{di_1}{dt} - \frac{n_2}{n_1} \frac{d\,(i_1 - i_{oo})}{dt} \right] \tag{12}$$

Mathematical Relations in the Secondary

A similar study can be made of the conditions in the secondary coil. It is evident that the e. m. f. induced by the exciting field must be

$$e_{i_2} = M \frac{di_1}{dt} - L_{o_2} \frac{di_2}{dt} \tag{13}$$

which may be written

$$e_{i_2} = L_{o_2} \left[\frac{n_1}{n_2} \frac{di_1}{dt} - \frac{di_2}{dt} \right] \tag{14}$$

or

$$e_{i_2} = L_{o_2} \left[\frac{n_1}{n_2} \frac{d(i_2 + i_{oo})}{dt} - \frac{di_2}{dt} \right]$$

as shown in (10) and (12) when considering the e. m. f. induced in the primary due to the same flux. The secondary terminal voltage, e_2, then is

$$e_2 = e_{i_2} - L_2 \frac{di_2}{dt} - i_2 r_2 \tag{15}$$

$$= M \frac{di_1}{dt} - L_{o_2} \frac{di_2}{dt} - L_2 \frac{di_2}{dt} - i_2 r_2 . \tag{16}$$

$$= L_{o_2} \left[\frac{n_1}{n_2} \frac{d(i_2 + i_{oo})}{dt} - \frac{di_2}{dt} \right] - L_2 \frac{di_2}{dt} - i_2 r_2 .$$

It also appears that

$$e_{i_2} = e_2 + i_2 r_2 + L_2 \frac{di_2}{dt} . \tag{17}$$

The constants used in the secondary correspond to the similar ones in the primary, both in their relative values and in their physical significance.

Combination of Equations through Ratio of Turns

That the electro-motive-force induced in the two coils by the same exciting flux should be in the ratio of the number of turns is perfectly evident. It may be checked from the equations evolved here if desired, by comparing

(14) with (10), bearing in mind the relation (9). Since this relation is true it is apparent that any voltage in either coil may be represented in equivalent terms with respect to the other coil by multiplying that voltage by the inverse ratio of turns. Then from this relation and (17), (7) may be written

$$e_1 = i_1 r_1 + L_1 \frac{di_1}{dt} + K_t e_{i_2}$$

$$= i_1 r_1 + L_1 \frac{di_1}{dt} + K_t i_2 r_2 + K_t L_2 \frac{di_2}{dt} + K_t e_2 \quad (18)$$

where $K_t = \frac{n_1}{n_2}$,

and the difference between the terminal equivalent voltages is

$$e_1 - K_t e_2 = i_1 r_1 + K_t i_2 r_2 + L_1 \frac{di_1}{dt} + K_t L_2 \frac{di_2}{dt}. \quad (19)$$

The Constants

The almost absolute constancy in value of r_1, r_2, L_1 and L_2 is noteworthy. Only a variation in temperature will change the resistances, r_1 and r_2 , and as tests and calculations are supposed to be made for "hot" temperatures, they may be considered constant. Since the iron core passes through both coils, no leakage line can have a complete iron path for its circuit but must bridge a considerable airgap. The effect of this high reluctance, constant permeability gap is to make the magnetization curve for this flux a comparatively straight line throughout a very great

range of density, assuring a similarly constant value for L. The limits of variation of the exciting current from a constant value or a constant times the induced e. m. f. have already been considered.

(Note:- The differential equation development as given in the preceding chapter is not according to standard practice in problems involving imperfect mutual induction, the case being treated rather as a combination of self-induction and perfect mutual induction in series. (See page 5) The inductive effect of the exciting field is separated from that of the leakage field because the m. m. f.'s of primary and secondary combine to keep the exciting field practically constant, whatever the load, while the leakage flux varies with the load. As appears later, this treatment makes it possible to eliminate the induced e. m. f. and the mutual or exciting flux except in the determination of the exciting current, and permits the study of the entire machine as a simple electric circuit.)

TRANSFORMER CHARACTERISTICS

The Constant Potential Machine

Transformers are designed to give varied characteristics according to the manner in which they are to be used. The most common type* is the constant potential transformer which receives energy from one circuit at a constant effective voltage and delivers it to a second circuit at as nearly constant an effective voltage as is possible. Thus far, the development has applied in a general way to all types. From here on, while much of the work will apply in the same way to the general transformer, it will be developed essentially for the constant potential machine. Constant potential should be construed to mean a constant wave shape of the same effective value and not a constant instantaneous voltage.

The Essential Characteristics

As with all machines, the first requirement of the transformer is that it be efficient. Due to the fact that modern lamps and motors operate to best effect only at voltages very near the values for which they are designed, the transformer must also have a very small percent regula-

*Other types are current transformers used mostly for instruments, constant current transformers for arc circuits, etc.

tion, that is the voltage delivered must be practically constant as the load varies from full load to no load. Again, since the regulation of the system is materially affected by the power factor, being worse for inductive load, the reactance of the transformer must be small enough so that the current will lag but little more on the primary side than it does in the secondary, -being inductive in itself, the machine cannot be made to improve the power factor, except for leading current.

The Solution; Methods of Attack

In order to determine the essential characteristics as noted above, it is sufficient to know the voltages and currents with their phase relations, at the primary and secondary terminals, for the various loads to be considered. The natural solution of the problem with constant impressed primary voltage would be to find the primary current by combining the secondary and exciting currents (equivalent values) and then, by substituting these and the primary voltage into (18), solve for the secondary voltage. The difficulties are that the secondary voltage must be known in order to determine the load current, and the induced e. m. f. in order to determine the exciting current. If the secondary voltage may be assumed, the secondary current can be found and from these values, with the constants r and L and the relation between the induced e. m. f. and the exciting current, the primary voltage and current can be determined. While this

makes the primary voltage dependent upon the secondary value and is therefore not natural, relative results determined in this way will be found to approximate normal test results very closely, in fact they will be fully as accurate as are the measured constants.

PREDETERMINATION OF CHARACTERISTICS:

SOLUTION BY DIFFERENTIAL EQUATION METHOD

General Solution

From the mathematical relations as previously established, the characteristics may now be predetermined. The secondary voltage and the constants of the circuits will be assumed as has just been suggested. The general solution is then as follows:

Let $e_2 = f_1$ function of time.

and let $i_2 = f_2$ function of time.

Then, by substitution into (17),

$$e_{i_2} = f_3 \ (t)$$

and from the development for (18)

$$e_{i_1} = K_t e_{i_2} = f_4 \ (t).$$

From (11a),

$$i_{oo} = K \ e_{i_1} = f_5 \ (t)$$

where K is determined by test,

and from (11)

$$i_1 = \frac{1}{K_t} i_2 + i_{oo} = f_6 \ (t).$$

Finally, by substitution into (7)

$$e_1 = f_7 \ (t).$$

The average power output is now

$$P = \frac{1}{T} \int_0^T e_2 \, i_2 \, dt$$

where T is the period of the wave, the average power input is

$$P_o = \frac{1}{T} \int_0^T e_1 \, i_1 \, dt$$

and the efficiency is

$$\text{Eff.} = \frac{P}{P_o} .$$

The effective values of voltage and current may be obtained in the ordinary way as from

$$E_{\text{eff.}} = \sqrt{\frac{2}{T} \int_{t_1}^{\frac{T}{2} + t_1} e^2 \, dt},$$

and from these and the values of average power the power factor may be determined,

$$PF = \frac{P}{E \, I} .$$

From an inspection of (19) and (11), it appears that for no load the equivalent primary and secondary voltages are practically equal. Thus, if the load be removed, the secondary voltage will change from e_2 or E_2 (effective) to $\frac{1}{K_t} e_1$ or $\frac{1}{K_t} E_1$ and the percent change will be the regulation, or

$$\% \text{ Reg.} = \frac{\frac{1}{K_t} E_1 - E_2}{E_2} \; 100.$$

Application to Sine Wave Conditions

By way of partial illustration of the solution, let

$$e_2 = \sqrt{2}\, E_2 \sin \omega t \qquad (20)$$

and

$$i_2 = \sqrt{2}\, I_2 \sin (\omega t + \alpha_2) \qquad (21)$$

where I_2 and α_2 are determined by the load. E_2 and I_2 are effective values of the secondary voltage and current respectively.

Substituting (20) and (21) into (17) and simplifying, the induced e. m. f. in the secondary is

$$e_{i_2} = \sqrt{2}\, E_{i_2} \sin (\omega t + \alpha), \qquad (22)$$

and, multiplying (22) by the ratio of turns, the primary induced e. m. f. is

$$e_{i_1} = K_t \sqrt{2}\, E_{i_2} \sin (\omega t + \alpha)$$

$$= \sqrt{2}\, E_{i_1} \sin (\omega t + \alpha). \qquad (23)$$

From test, when

$$e_i' = \sqrt{2}\, E_i' \sin \omega t$$

is impressed upon the primary (secondary open circuited) or when

$$-e_i' = -\sqrt{2} E_i' \sin \omega t$$

is induced, the exciting current has been found to be

$$i_{oo}' = I_{oo}' \sin (\omega t + \alpha')$$

where α' is the angle by which this current is out of phase

with the impressed voltage. This equation is an approximation but is sufficiently accurate where final results are to be obtained in terms of effective values.* Then for the induced voltage in (23), the exciting current is

$$i_{oo} = -\sqrt{2}\, K\, E_{i_1} \sin(\omega t + \alpha + \alpha')$$

$$= -\sqrt{2}\, I_{oo} \sin(\omega t + \alpha_{oo}). \qquad (24)$$

Aside from the exciting current, the primary must supply a current to overcome the counter m. m. f. from the secondary load current. This is

$$-\frac{1}{K_t} i_2$$

and the total primary current is then

$$i_1 = i_{oo} - \frac{1}{K_t} i_2$$

$$= -\sqrt{2} I_1 \sin(\omega t + \alpha_1). \qquad (25)$$

The primary voltage may now be determined by substituting (24) and (25) into (7) and simplifying, or

$$e_1 = -\sqrt{2}\, E_1 \sin(\omega t + \beta), \qquad (26)$$

the negative sign in (26) showing that, when $\beta = 0$, e_1 will be 180° out of phase with e_2.

From the mathematics of the sine wave, the final results can now be obtained as follows:

$$\text{Effective value of } e_2 = \frac{\sqrt{2}\, E_2}{\sqrt{2}} = E_2.$$

*See page 10

Effective value of $i_2 = \frac{\sqrt{2}\ I_2}{\sqrt{2}} = I_2$.

Effective value of $e_1 = E_1$

Effective value of $i_1 = I_1$

Phase angle between e_2 and $i_2 = \alpha_2$.

Phase angle between e_1 and $i_1 = (\beta - \alpha_1)$.

$$\text{Output} = P = \frac{1}{T}\int_0^T e_2\ i_2\ dt = E_2\ I_2 \cos \alpha_2 .$$

$$\text{Input} = P_0 = \frac{1}{T}\int_0^T e_1\ i_1\ dt = E_1\ I_1 \cos (\beta - \alpha_1).$$

$$\text{Efficiency} = \frac{E_2\ I_2 \cos \alpha_2}{E_1\ I_1 \cos (\beta - \alpha_1)} .$$

$$P\ F_2 = \frac{E_2\ I_2 \cos \alpha_2}{E_2\ I_2} = \cos \alpha_2 .$$

$$P\ F_1 = \cos (\beta - \alpha_1).$$

$$\text{Regulation} = \frac{\frac{1}{K_t} E_1 - E_2}{E_2} .$$

The calculation where sine waves of secondary voltage and current can be assumed is thus seen to be comparatively simple, the only difficulty appearing in the trigonometric transformations necessary in combining the various waves. It will later be shown that, by using the general number and by resolving all waves into component sine and cosine waves with respect to a given zero angle, much of the labor of the ordin-

ary analytical method can be dispensed with. Where the waves are not sine waves, no such short cut method can be resorted to; furthermore it is probable that the assumptions made regarding i_{oo} would then have to be modified, and the problem thus made more difficult. It is therefore fortunate that at least equivalent sine waves can be used in practically all cases except where instantaneous or transient conditions are under consideration.

GRAPHICAL SOLUTION BY WAVES AND BY VECTOR DIAGRAMS

When desirable for clearness or for checking, the relations just expressed analytically may be shown graphically by waves as in figures 1, 2 and 3. Also where the sine wave may be assumed, the waves may be represented by equivalent systems of vectors, as in figures 4, 5 and 6. In each case in the diagrams the secondary voltage and current and the power factor of the receiving circuit have been assumed. (See pages 25 to 28 for diagrams.)

The graphical methods really need no explanation. For the sake of clearness a ratio of 1:1 has been chosen and the resistance and reactance drops have been assumed to be about ten times normal in all but figure 3 and figure 6, showing inductive load. In these, actual values for a small lighting transformer are shown so as to illustrate real conditions. The induced voltages are shown as E_{i_1} and E_{i_2} while that part of the impressed primary voltage which is consumed by E_{i_1} is shown as E_{i_o}. A normal phase angle has been chosen between E_{i_o} and I_{oo} and the magnitude of I_{oo} has been permitted to vary with E_{i_o}. I_{oo}, also, is shown too large in all but figure 3 and figure 6. Even by considering all voltages and currents as percentage quantities with reference to normal full-load non-inductive values in each coil, so that the ratio will appear as 1:1,

it is evident that errors of measurement will make the graphical methods unsatisfactory, except as mere expressions of physical relations. The useful flux, φ, is shown in the vector diagrams, leading the induced e. m. f. by 90° .

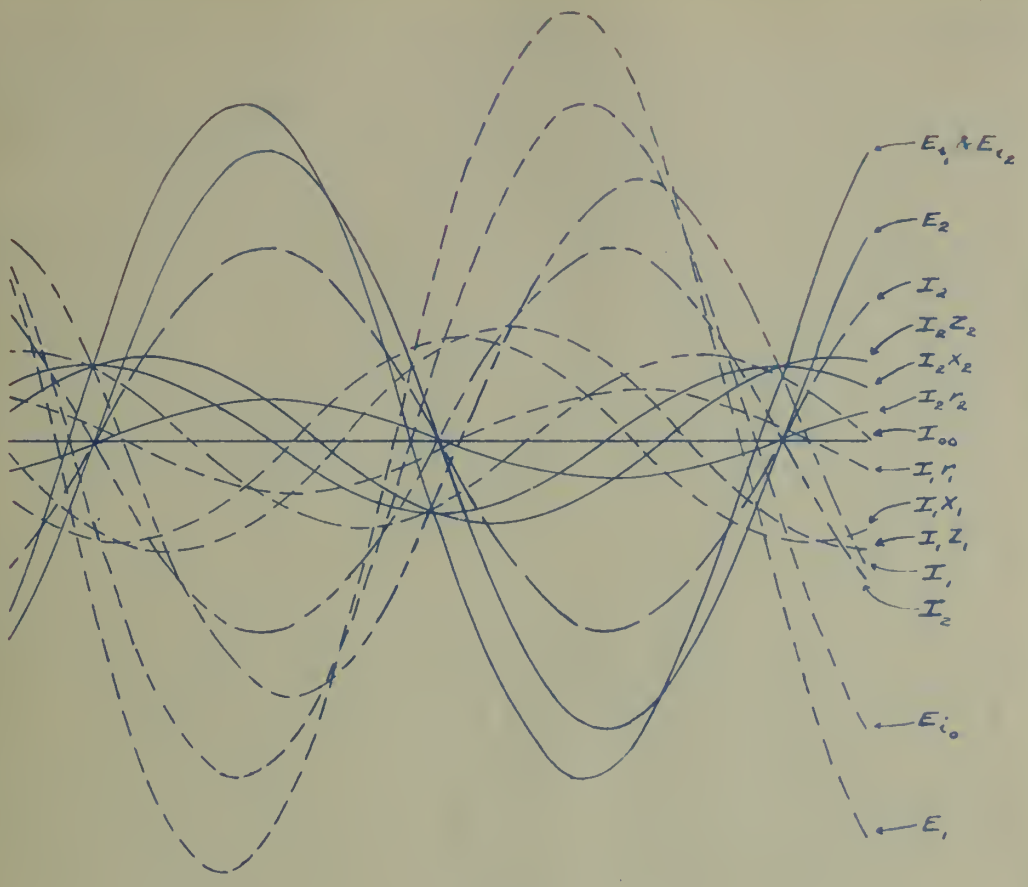

Fig. 1.

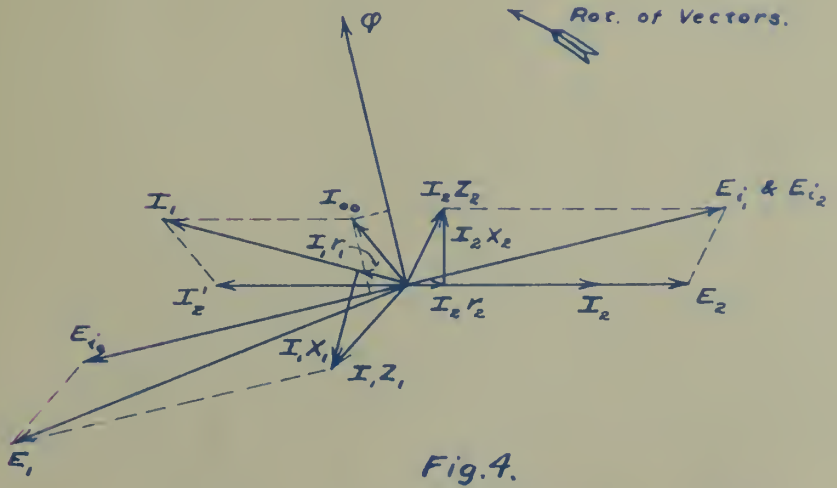

Fig. 4.

System of Waves and Vector Diagram: Non-Inductive Load.

$r_1 = r_2 = 13.3\%$. $X_1 = X_2 = 26.7\%$. $I_{cl} = 20\%$. $I_{mag} = 40\%$.

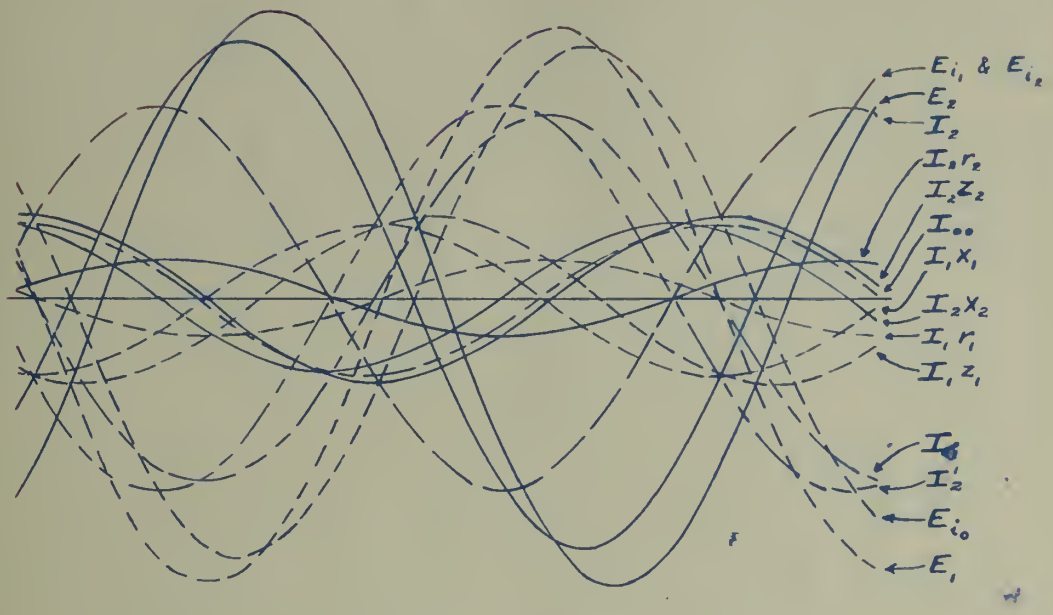

Fig. 2.

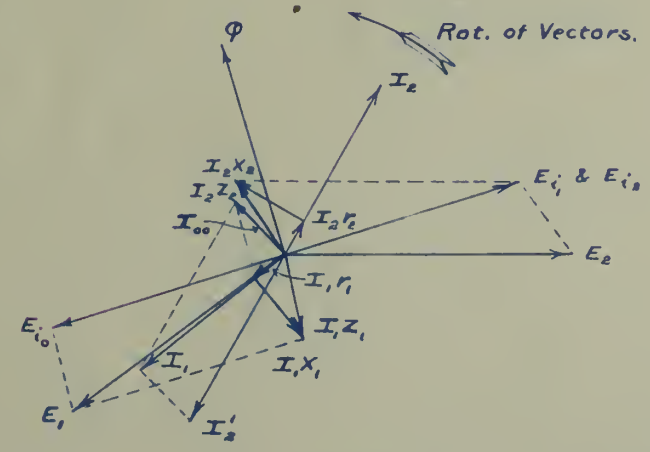

Fig. 5.

System of Waves and Vector Diagram: Capacity Load, PF = .5.

$r_1 = r_2 = 13.3\%$. $X_1 = X_2 = 26.7\%$. $I_{cu} = 20\%$. $I_{mag} = 40\%$.

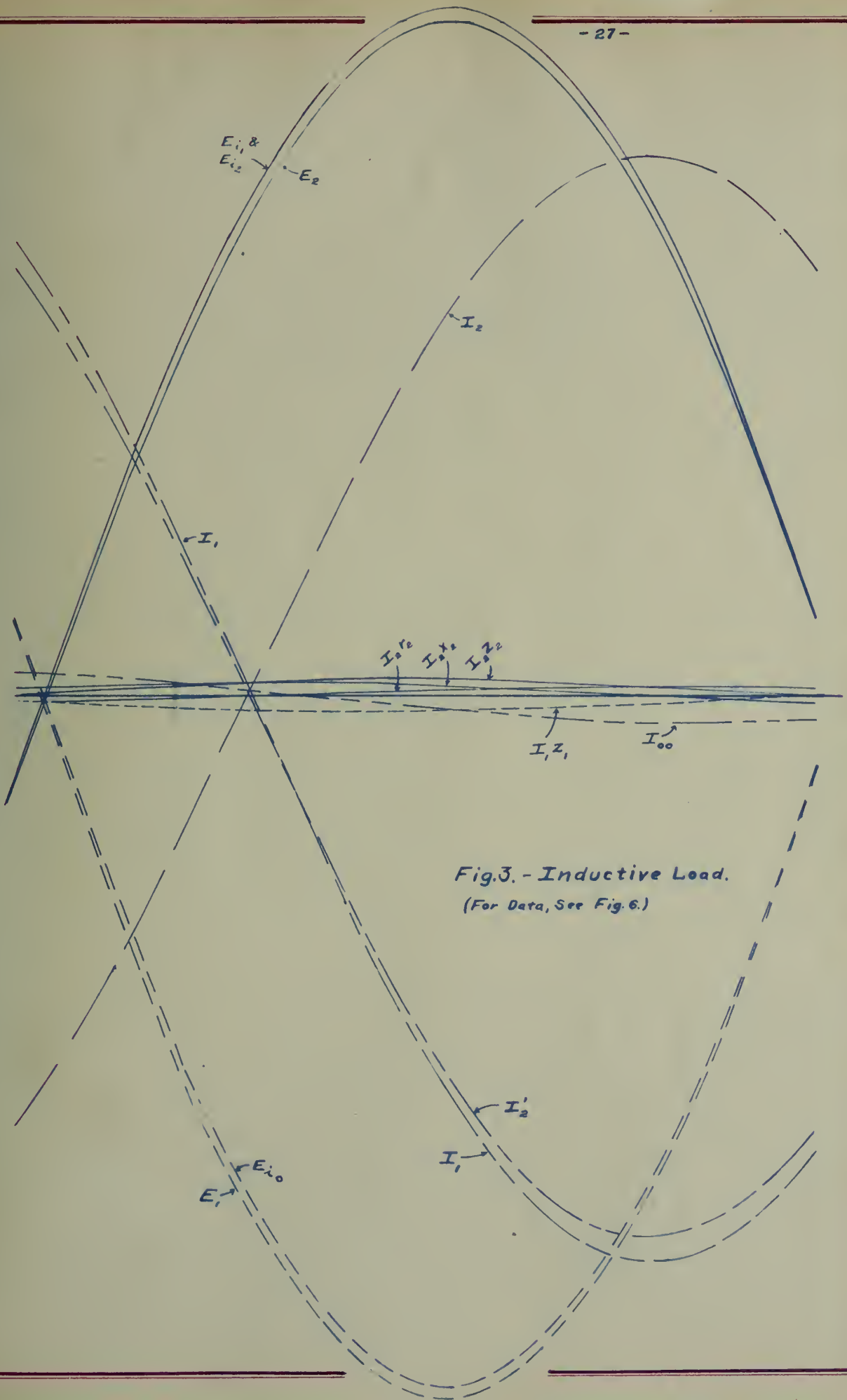

Fig. 3. - Inductive Load.
(For Data, See Fig. 6.)

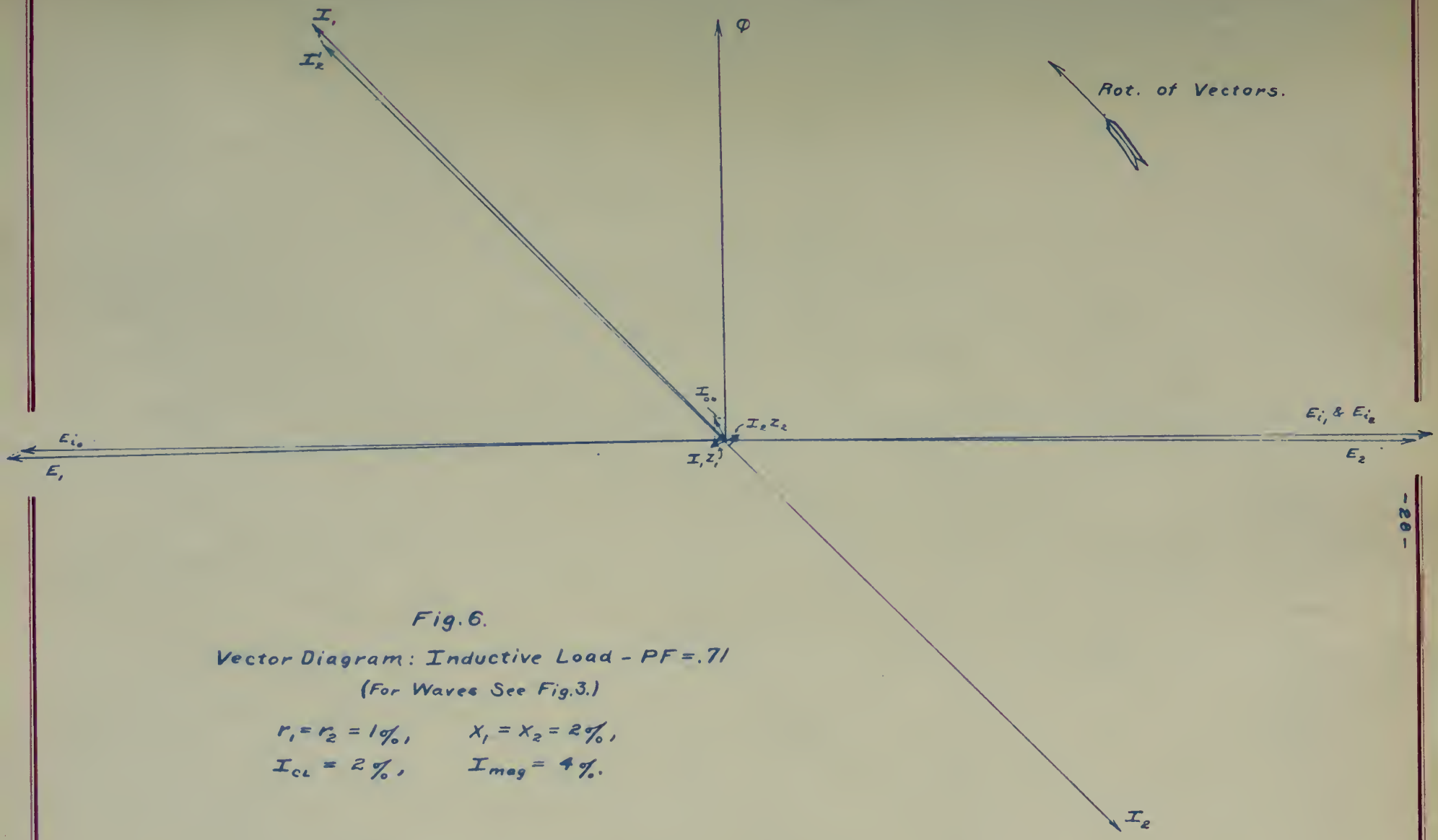

Fig. 6.

Vector Diagram: Inductive Load - PF = .71

(For Waves See Fig. 3.)

$r_1 = r_2 = 1\%$, $X_1 = X_2 = 2\%$,

$I_{CL} = 2\%$, $I_{mag} = 4\%$.

SOLUTION BY COMPLEX QUANTITIES

Substitution of General Numbers for Vectors

There can be no doubt about the distinct advantage of graphical methods over analytical, from the standpoint of clearness, but they are at best very inaccurate and it is necessary to return to mathematical expressions for dependable results. It has already been demonstrated that the combination or solution of a system of waves by means of the equations representing them is likely to be a long and involved process. It is fortunate, therefore, that a more simple method is available in the mathematics of the general number which may be applied directly to the vector diagram representing this system of waves. The general number method has the added advantage of the vector diagram over the waves; that of clearness and ease of manipulation. It is, however, limited to the conditions where the vector diagram can be used and is therefore applicable only in those cases where sine waves are found or may be substituted for the waves in the original system.

The general number, $a_1 + j\, a_2$, or as is usually given in algebra*, $a_1 + a_2\, i$, represents a line or vector

*Because of the universal use of i for current in electrical calculations, j is generally used in place of the mathematical i.

(See Fig. 7) of length

$$A = \sqrt{a_1^2 + a_2^2},$$

extending from the origin into the first quadrant and making an angle α with the horizontal axis, where

$$\tan \alpha = \frac{a_2}{a_1} .$$

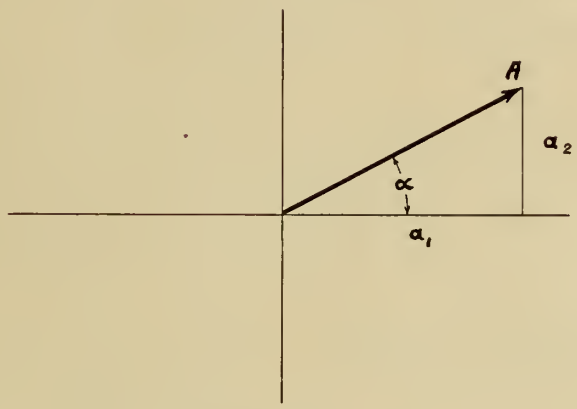

Fig. 7.

The general number may be represented by its scalar length with a dot, as

$$\dot{A} = a_1 + j\, a_2 .$$

The line represented by $\dot{A}$ may also be described as a vector of length A, rotated through a positive angle from the horizontal axis, the mathematical operation of doing this being expressed by the operator or coefficient of A in

$$\dot{A} = A (\cos \alpha + j \sin \alpha).$$

Rotation into the fourth quadrant would be through an angle $(-\alpha)$ and such a vector would be

$$\dot{B} = B\left[\cos(-\alpha) + j\sin(-\alpha)\right]$$
$$= B(\cos\alpha - j\sin\alpha)$$
$$= b_1 - j\,b_2.$$

where $$\tan\alpha = \frac{-b_2}{b_1}.$$

Note that the sign preceding the j indicates direction in the vertical or j axis, but that, when discussing the components of the vector, it is desirable to retain this sign as a coefficient of the component.

It is interesting to note that the operators given above are reciprocals of each other, or

$$\cos\alpha - j\sin\alpha = \frac{1}{\cos\alpha + j\sin\alpha}$$

so that if α is the same for $\dot{A}$ and $\dot{B}$ and if $B = \frac{1}{A}$, then

$$\dot{B} = \frac{1}{\dot{A}} = \frac{1}{A(\cos\alpha + j\sin\alpha)}$$

$$= \frac{1}{A}(\cos\alpha - j\sin\alpha).$$

No attempt will be made here to explain more in detail the the general application of the complex quantity to the vector diagram, that being done in many texts written by Dr. Steinmetz and others and in many magazine articles. The direct application to alternating current problems will be considered very briefly for the same reasons.

Application of General Numbers to Alternating Current Problems

By definition, the impedance of a circuit is that factor by which the current is multiplied to determine the total drop across the circuit, or

$$Z = \frac{E}{I}.$$

If α is the angle of phase difference between $\dot{E}$ and $\dot{I}$ and and the current leads* the horizontal axis by θ, then

$$\dot{E} = E\left[\cos(\theta + \alpha) + j \sin(\theta + \alpha)\right],$$

$$\dot{I} = I\ (\cos\theta + j \sin\theta)$$

and

$$\dot{Z} = \frac{\dot{E}}{\dot{I}} = \frac{E}{I}(\cos\alpha + j \sin\alpha)$$

$$= \frac{E\cos\alpha}{I} + j\frac{E\sin\alpha}{I}$$

$$= r + j\,x.$$

Also by definition, the admittance of a circuit is the factor by which the voltage impressed is multiplied to obtain the current flowing, or

$$Y = \frac{I}{E}$$

and vectorially, using the above values of voltage and current,

$$\dot{Y} = \frac{\dot{I}}{\dot{E}}$$

*By leading is meant separation by a positive angle in the direction of mathematical rotation or counter-clockwise.

$$= \frac{I\ (\cos\theta + j \sin\theta\ )}{E\ \left[\cos\ (\theta + \alpha) + j \sin\ (\theta + \alpha)\right]}$$

$$= \frac{I}{E}\ (\cos\alpha + j\ (-\sin\alpha))$$

$$= g + j\ b.$$

Again

$$\dot{Y} = \frac{1}{\dot{Z}}$$

$$= \frac{1}{r + j\ x}$$

$$= \frac{r}{r^2 + x^2} + j\ \frac{-x}{r^2 + x^2}$$

$$= g + j\ b.$$

Thus it is seen that the conductance or in phase component of the admittance is

$$g = \frac{r}{r^2 + x^2}$$

and the susceptance or quadrature component is

$$b = \frac{-x}{r^2 + x^2}$$

where r and x are the resistance and reactance respectively. Some writers prefer

$$b = \frac{x}{r^2 + x^2}$$

and

$$\dot{Y} = g - j\ b.$$

This is satisfactory for special cases but where the problem is perfectly general and the sign of b is not known, it

is an advantage to develop all the literal mathematics for positive quantities so that, when numerical values are substituted, the signs in the results will immediately determine their physical relations. This plan is followed closely in this thesis. Another standard form adopted is the use of a prime (') to indicate the j component of voltage or current, permitting the use of all subscripts to designate the various parts of the circuits.

As a result of the relations just discussed, there are four standard general numbers, representing the four vectors or complex quantities used in electrical calculations, namely:

$$\dot{E} = e + j\,e' \qquad \text{where } E = \sqrt{e^2 + e'^2}.$$

$$\dot{I} = i + j\,i' \qquad \text{where } I = \sqrt{i^2 + i'^2}.$$

$$\dot{Z} = r + j\,x \qquad \text{where } X = \sqrt{r^2 + x^2}.$$

$$\dot{Y} = g + j\,b \qquad \text{where } Y = \sqrt{g^2 + b^2}.$$

The mathematics and diagrams for a circuit whose total impedance is

$$\dot{Z} = r + j\,x$$

and where either the current or the voltage is known, are as follows.

Current Known

Let $\dot{I} = i + j\,i'$ be the current flowing, referred to i as reference vector.

Then

$$\dot{E} = \dot{I}\,\dot{Z}$$

$$= (i + j\, i')(r + jx)$$

$$= (i\, r - i'\, x) + j\,(i'\, r + i\, x)$$

$$= e + j\, e' ,$$

where the vectors and their components are as shown in figures 8 and 9. (See page 36.)

Voltage Known

Let $\dot{E} = e + j\, e'$ be the voltage impressed.

Then

$$\dot{I} = \frac{e + j\, e'}{\dot{Z}}$$

$$= (e + j\, e')\, \dot{Y}$$

$$= (e + j\, e')\,(g + j\, b)$$

$$= (e\, g - e'\, b) + j(e'\, g + e\, b)$$

$$= i + j\, i'.$$

Mathematically this solution is complete and general. To interpret it for a special case, the signs of g and b must be determined from the physics of the problem. To make the diagrams in figures 10 and 11 conform to those in figures 8 and 9, the same physical relations must exist, or g must be positive and b negative because r and x are both positive. For these diagrams then, the numerical result may be written

$$\dot{I} = \left[e\, g - e'\,(-b)\right] + j\,\left[e'\, g + e\,(-b)\right]$$

$$= (e\, g + e'\, b) + j\,(e'\, g - e\, b),$$

from which figures 10 and 11 are obtained.

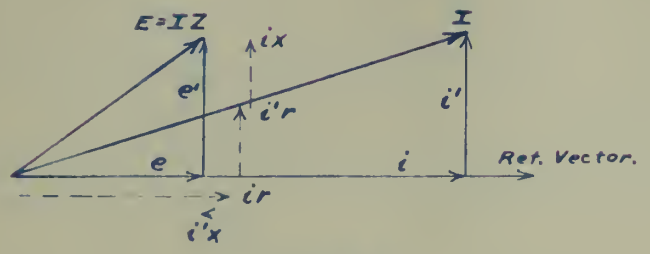

Fig. 8.

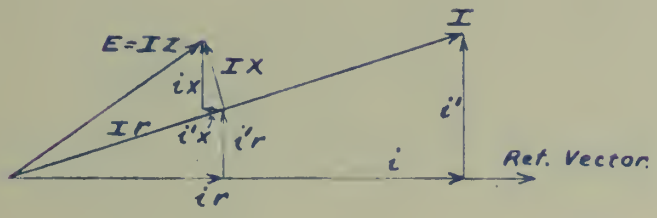

Fig. 9.

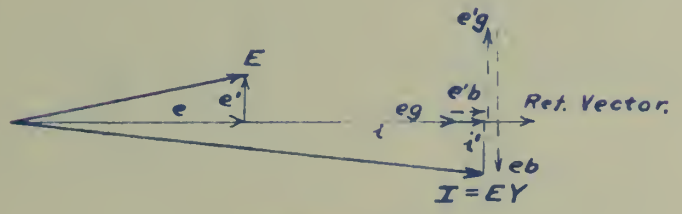

Fig. 10.

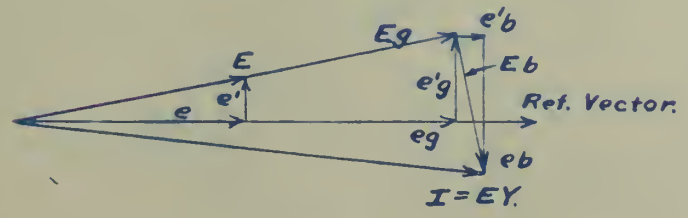

Fig. 11.

Equation for Power

The power in any circuit is

$$P = E\ I \cos \alpha$$

where α is the angle of phase difference between the voltage E and the current I. When the voltage and current are given as

$$\dot{E} = e + j\ e'$$

and

$$\dot{I} = i + j\ i'$$

the power equation may also be written

$$P = e\ i + e'\ i'.$$

This can be proven as follows. The power is given as

$$P = E\ I \cos \alpha$$

$$= E\ I \cos (\alpha_1 - \alpha_2) \qquad (a)$$

where $\alpha_1 = \text{arc cos } \frac{e}{E}$ and $\alpha_2 = \text{arc cos } \frac{i}{I}$.

Expanding (a) and collecting terms,

$$P = E\ I\ (\cos \alpha_1 \cos \alpha_2 + \sin \alpha_1 \sin \alpha_2)$$

$$= E \cos \alpha_1\ I \cos \alpha_2 + E \sin \alpha_1\ I \sin \alpha_2$$

$$= e\ i + e'\ i'. \qquad \text{Q. E. D.}$$

Application to the Transformer

From the physical relations as explained for the transformer it will be seen that the machine may be represented by an equivalent circuit as shown in figure 12. Either percentage calculation or a 1:1 ratio is assumed. The admittance $\dot{Y}_{oo}$ is the factor by which to multiply the voltage consumed by the induced e. m. f., to obtain the exciting current in its proper magnitude and phase relation with that voltage.

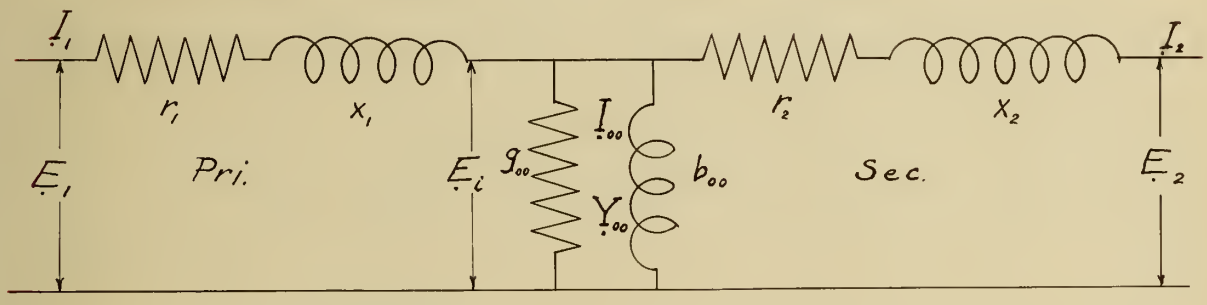

Fig. 12

Let the constants, voltages and currents be as indicated in figure 12 and let $\dot{E}_2$ be the reference vector. Then

$$\dot{E}_2 = e_2 + j\,0.$$

Also let the secondary current required by the load be

$$\dot{I}_2 = i_2 + j\,i'_2 .$$

Then the impedance drop in the secondary is

$$\dot{I}_2\,\dot{Z}_2 = (i_2 + j\,i'_2)(r_2 + j\,x_2)$$

$$= (i_2 r_2 - i'_2 x_2) + j (i'_2 r_2 + i_2 x_2)$$

and the induced voltage is

$$\dot{E}_i = \dot{E}_2 + \dot{I}_2 \dot{Z}_2$$

$$= (e_2 + i_2 r_2 - i'_2 x_2) + j (i'_2 r_2 + i_2 x_2)$$

$$= e_i + j e'_i .$$

While the primary voltage consumed by this induced voltage is really 180° out of phase with it, as seen in figure 6, it is not necessary to encumber the mathematics with these negative signs, if the fact that all primary vectors are opposite in sign to the corresponding secondary vectors is borne in mind. Then the voltage $\dot{E}_i$ may be considered as impressed upon the exciting admittance, and the exciting current is

$$\dot{I}_{oo} = \dot{E}_i \dot{Y}_{oo}$$

$$= (e_i + j e'_i)(g_{oo} + j b_{oo})$$

$$= (e_i g_{oo} - e'_i b_{oo}) + j (e'_i g_{oo} + e_i b_{oo})$$

$$= i_{oo} + j i'_{oo}.$$

The primary current is the sum of the secondary and exciting currents considered in their phase relations, or

$$\dot{I}_1 = \dot{I}_2 + \dot{I}_{oo}$$

$$= (i_2 + j i'_2) + (i_{oo} + j i'_{oo})$$

$$= (i_2 + i_{oo}) + j (i'_2 + i'_{oo})$$

$$= i_1 + j i'_1.$$

Then the primary drop is

$$\dot{I}_1 \dot{Z}_1 = (i_1 + j\, i'_1)(r_1 + j\, x_1)$$

$$= (i_1 r_1 - i'_1 x_1) + j\,(i'_1 r_1 + i_1 x_1)$$

and the primary impressed voltage is

$$\dot{E}_1 = \dot{E}_1 + \dot{I}_1 \dot{Z}_1$$

$$= (e_i + i_1 r_1 - i'_1 x_1) + j\,(e'_i + i'_1 r_1 + i_1 x_1)$$

$$= e_1 + j\, e'_1.$$

Thus from the load voltage and current, the groupings as shown above and the constants of the machine as determined from test, the following values have been obtained:

	Vectorial	Scalar
Assumed	$\dot{E}_2 = e_2 + j\, o$	$E_2 = \sqrt{e_2^2 + e'^2_2} = e_2$
Assumed	$\dot{I}_2 = i_2 + j\, i'_2$	$I_2 = \sqrt{i_2^2 + i'^2_2}$
	$\dot{I}_{oo} = i_{oo} + j\, i'_{oo}$	$I_{oo} = \sqrt{i_{oo}^2 + i'^2_{oo}}$
	$\dot{I}_1 = i_1 + j\, i'_1$	$I_1 = \sqrt{i_1^2 + i'^2_1}$
	$\dot{E}_1 = e_1 + j\, e'_1$	$E_1 = \sqrt{e_1^2 + e'^2_1}$

From these values and the following formulas, it is now possible to obtain the regulation, the power factor at primary and secondary terminals and the efficiency.

$$\text{Regulation} = \frac{E_1 - E_2}{E_2}.$$

$$\text{Watts Output} = P = e_2\, i_2 + e'_2\, i'_2 = e_2\, i_2$$

$$\text{Voltamperes Output} = E_2\, I_2$$

$$\text{Power factor of Load} = P\,F = \frac{e_2\, i_2}{E_2\, I_2} = \frac{i_2}{I_2}$$

$$\text{Watts Input} = P_o = e_1\, i_1 + e'_1\, i'_1$$

$$\text{Voltamperes Input} = E_1\, I_1$$

$$\text{Power factor at Intake} = P\,F_o = \frac{e_1\, i_1 + e'_1\, i'_1}{E_1\, I_1}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{e_2\, i_2}{e_1\, i_1 + e'_1\, i'_1} \quad .$$

To obtain the operating characteristics for a definite machine under a specific load it is merely necessary to substitute the proper numerical values in the above expressions. Such calculations are shown in the appendix for varying load at unity power factor and leading and lagging load current and for varying power factor at several loads. Since the impedances of the primary and secondary coils and of the exciting branch are inductive, the numerical test values to be substituted for r, x and g should be positive and that for b should be negative. The sign of i' is fixed by the character of the load, positive for leading current and negative for lagging current.

Complete Vector Diagrams, Showing Components

Representative diagrams for leading and lagging load currents are given in figures 13 and 14, showing all the vectors and their components. Ratios of 1:1 are

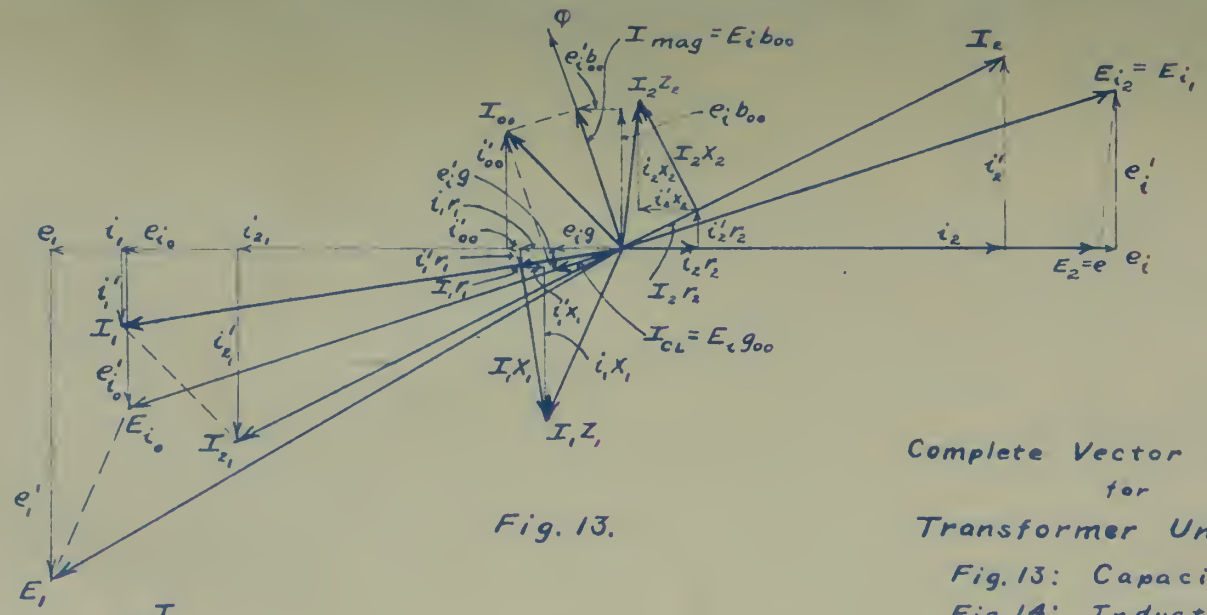

Fig. 13.

Complete Vector Diagrams
for
Transformer Under Load.

Fig. 13: Capacity Load.
Fig. 14: Inductive Load.

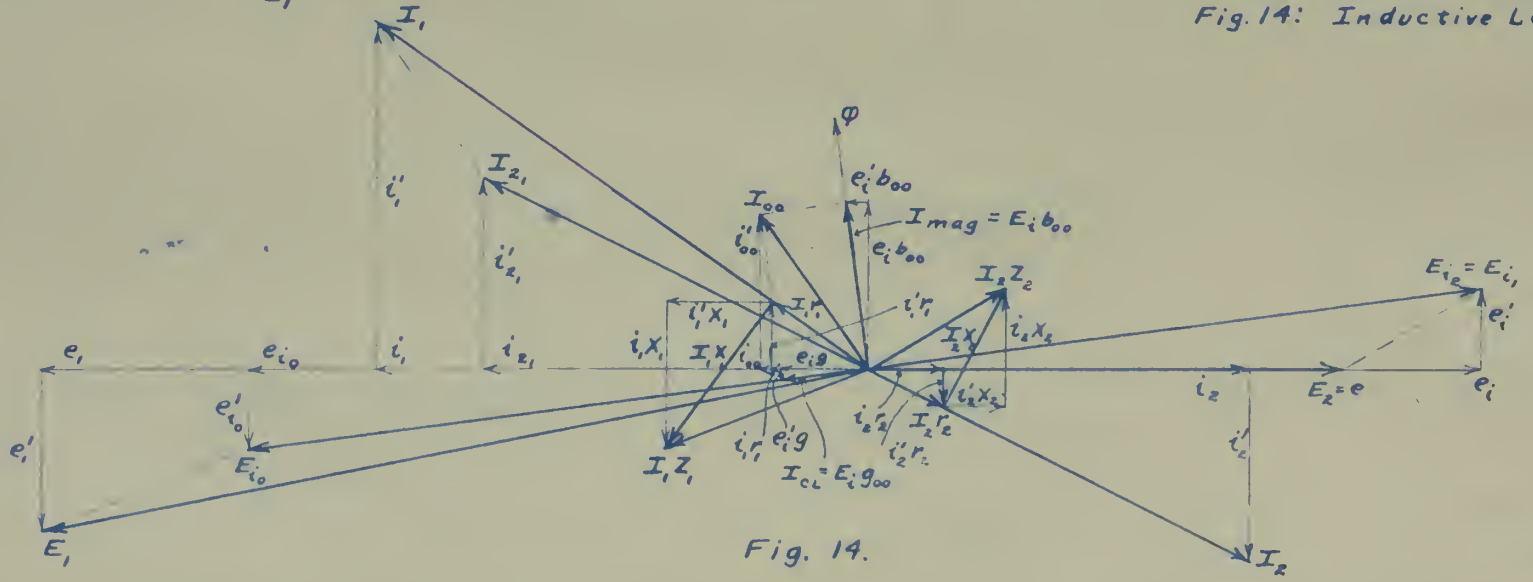

Fig. 14.

assumed. The symbols are as already used, except that φ represents only the useful or exciting flux and that I_{2_1} is that part of the primary current which overcomes the counter m. m. f. of the secondary or load current. The diagrams are exaggerated so as to show method and comparative results rather than exact magnitudes and phase relations.

SUMMARY

The purpose of this thesis has been two-fold, first to explain the physical relations existing in the transformer by means of mathematical expressions and then to develop from these expressions more or less convenient methods for predetermining the operating characteristics of the machine. In describing the transformer, nothing more complicated than very simple differentials has been encountered. Equations like (6), (12) and (18) have been developed in which definite relations are shown to exist between the several voltages and currents. The only approximation has been that the exciting current varies directly with the voltage induced by the useful field and bears a constant phase relation with the same, the accuracy of this approximation being discussed on page 10.

In applying this mathematics, the specific cases have been divided into two classes, one including all cases where sine waves of voltage and current may be assumed and the other including those where this assumption is not possible. The solution of problems of the second class is likely to be long and involved, but where sine waves may be used, the solution is comparatively simple, even when performed analytically or by means of the differential equations. Also, since sine waves lend themselves readily to graphical

representation by curves or by vectors, problems of this class may be solved graphically. The chief advantages of the graphical method are clearness both in showing the physical relations in the problem and in comparing the results when certain of these relations are varied. (See figures 4, 5 and 6.) The disadvantage is the inaccuracy of the results.

The complex quantity or general number method, where the vectors are represented analytically by general numbers, combines the clearness of the graphical method with the accuracy of the purely analytical method. Like the graphical, it can be used only where sine waves may be considered. It has an additional advantage in the ease with which constants or loads may be varied in the calculation without greatly increasing the labor of solving beyond that for the single set of values. (See Appendix.)

In closing it should be said that the conditions as set forth in the problem and the accuracy desired in the results must determine the method to be used. It is fortunate, however, that sine waves are approximated in most cases and may be assumed, and where this is possible the complex quantity method of calculation is recommended.

APPENDIX

CALCULATIONS BY THE COMPLEX QUANTITY METHOD

Method of Determination of Constants from Tests

The constants used in the transformer calculations are the primary and secondary impedances and the exciting admittance, and are readily obtained from the ordinary resistance, short circuit and open circuit tests. The short circuit data gives at once the equivalent impedance of the machine from which the resistance may be taken leaving the total equivalent reactance, half of which is allotted each winding. The admittance in the open circuit test is the proportionality factor between the voltage and the exciting current, and may be separated into conductance and susceptance, or if the exciting current has first been separated into the coreloss and magnetizing currents the conductance and susceptance may be obtained directly from these values and the voltage. The constants as thus determined are expressed in their ordinary units, -ohms and mhos.

The problem is simplified if reference quantities of primary and secondary voltage, current and power are chosen and the constants expressed as factors having reference to these quantities. Percent impedance would thus be defined as the percent drop with respect to 100% voltage

which would be consumed if 100% current is passed through the impedance, and percent admittance would be defined in a similar way except that the current due to 100% voltage is here taken in terms of 100% current. It is convenient to choose the rated values as reference quantities in the secondary, and the same values when corrected by the ratio of turns as those in the primary. It should be noted that 100% voltage and current output will give 100% power output only when the power factor is unity.

Specifically, then, the percent constants may be determined by substitution into the following expressions.

$$\% r_2 = \frac{100\ (100\%\ I_2)\ r_2}{(100\%\ E_2)},$$

$$\% x_2 = \frac{100\ (100\%\ I_2)\ x_2}{(100\%\ E_2)},$$

$$\% r_1 = \frac{100\ (100\%\ I_1)\ r_1}{(100\%\ E_1)},$$

$$\% x_1 = \frac{100\ (100\%\ I_1)\ x_1}{(100\%\ E_1)},$$

$$\% g = \frac{100\ (100\%\ E)\ g}{100\%\ I},$$

$$\% b = \frac{100\ (100\%\ E)\ b}{100\%\ I}.$$

In determining percent g and b, the reference voltage and current to be used are those for the winding on which the data was taken, -usually the secondary. When normal voltage is used in the test, the numerators are the coreloss

and magnetizing currents and the constants are

$$\% g = \frac{100 \cdot I_{C_L}}{(100\% \ I)}$$

and

$$\% b = \frac{100 \cdot I_{mag.}}{(100\% \ I)}.$$

The short circuit or impedance drop may be considered in the same way if the current under test is normal.

Numerical Constants from Test Data

Consider a transformer with the rating

50 K. V. A., 2200:220 volts, 60 cycles,

on which the following test data is assumed to have been taken:

Resistance,-

$$R_p = .97 \ , \quad R_s = .0097.$$

Short circuit, voltage applied to primary,-

$$E_p = 98.5 \ , \quad I_p = 22.7 = \text{normal}.$$

Open circuit, voltage applied to secondary,-

$$E_s = 220 \ , \quad I_{oo} = 12.25, \quad \text{Watts} = 1000.$$

The percent resistances are therefore

$$r_p = \frac{100 \cdot 22.7 \cdot .97}{2200} = 1\% = .01 \text{ in percent.}$$

$$r_s = \frac{100 \cdot 227 \cdot .0097}{220} = 1\% = .01 \text{ in percent.}$$

The percent impedance is directly obtained from the test data,

$$Z = \frac{100 \cdot 98.5}{2200} = 4.48\% = .0448 \text{ in percent.}$$

The total equivalent resistance is .02, hence the total equivalent reactance is

$$x = \sqrt{.0448^2 - .02^2} = .04 \text{ in percent}$$

and the primary and secondary reactances are,

$$x_p = x_s = \frac{x}{2} = .02 \text{ in percent.}$$

Thus the primary and secondary impedances in percent and complex quantities are

$$\underset{\cdot}{Z}_p = r_p + j\, x_p = .01 + j\, .02,$$

and

$$\underset{\cdot}{Z}_s = r_s + j\, x_s = .01 + j\, .02.$$

From the open circuit data, the coreloss current is

$$I_{CL} = \frac{W}{E} = \frac{1000}{220} = 4.55 \text{ amperes}$$

and the magnetizing current is

$$I_m = \sqrt{I_{oo}^2 - I_{CL}^2}$$

$$= \sqrt{12.25^2 - 4.55^2} = 11.35 \text{ amperes.}$$

The percent conductance and susceptance are now directly obtained as

$$g = \frac{100 \cdot 4.55}{227} = 2\% = .02 \text{ in percent}$$

and

$$b = \frac{-100 \cdot 11.35}{227} = -5\% = -.05 \text{ in percent.}$$

These values also represent the percent coreloss and magnetizing currents respectively.

CALCULATIONS

The literal solution of the transformer is given on pages 38 to 44 in the body of the thesis. All that is necessary then, in the calculation under any condition of loading, is that the proper constants and load values be substituted into the equations and the indicated operations performed. Two sets of calculations are given here, one showing the effect upon the efficiency, power factor, regulation etc. of the character of the load and the other showing the effect upon the same values of a variation in the inherent constants of the machine under consideration. Each column in each table is really a complete problem in itself, determining the characteristics for one set of constants and one condition of load.

CALCULATION OF THE TRANSFORMER

Effect of the Variation of the Power Factor and Amount of Load Upon its Operating Characteristics.

	PF	1	1	1	1	.8	.8 lag	.8	.8	.8	.8 lead	.8	.8
	e	1	1	1	1	1	1	1	1	1	1	1	1
Consts.	$r_p = r_s$	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
	$x_p = x_s$	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
	g	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
	b	-.05	-.05	-.05	-.05	-.05	-.05	-.05	-.05	-.05	-.05	-.05	-.05
I	I	0	.5	1	1.5	0	.5	1	1.5	0	.5	1	1.5
	i	0	.5	1	1.5	0	.4	.8	1.2	0	.4	.8	1.2
	i'	0	0	0	0	0	-.3	-.6	-.9	0	.3	.6	.9
E_i	ir_s	0	.005	.01	.015	0	.004	.008	.012	0	.004	.008	.012
	$i'x_s$	0	0	0	0	0	-.006	-.012	-.018	0	.006	.012	.018
	$e_i = e + ir_s - i'x_s$	1	1.005	1.01	1.015	1	1.01	1.02	1.03	1	.998	.996	.994
	$i'r_s$	0	0	0	0	0	-.003	-.006	-.009	0	.003	.006	.009
	ix_s	0	.01	.02	.03	0	.008	.016	.024	0	.008	.016	.024
	$e_i' = i'r_s + ix_s$	0	.01	.02	.03	0	.005	.010	.015	0	.011	.022	.033
I_{oo}	$e_i g$	.02	.0201	.0202	.0203	.02	.0202	.0204	.0206	.02	.0200	.0199	.0198
	$e_i' b$	0	-.0005	-.001	-.0015	0	-.0002	-.0005	-.0007	0	-.0005	-.0011	-.0017
	$i_{oo} = e_i g - e_i' b$	.02	.0206	.0212	.0218	.02	.0204	.0209	.0213	.02	.0205	.0210	.0215
	$e_i' g$	0	.0002	.0004	.0006	0	.0001	.0002	.0003	0	.0002	.0004	.0007
	$e_i' b$	-.05	-.0502	-.0505	-.0507	-.05	-.0505	-.0510	-.0515	-.05	-.0499	-.0498	-.0491
	$i_{oo}' = e_i' g + e_i b$	-.05	-.05	-.0501	-.0501	-.05	-.0504	-.0508	-.0512	-.05	-.0497	-.0494	-.0490
I_p	$i_p = i + i_{oo}$	.02	.5206	1.021	1.522	.02	.4204	.821	1.221	.02	.4205	.821	1.221
	$i_p' = i' + i_{oo}'$	-.05	-.05	-.05	-.05	-.05	-.3504	-.651	-.951	-.05	.250	.551	.851
	$I_p = \sqrt{i_p^2 + i_p'^2}$	.054	.523	1.022	1.523	.054	.547	1.046	1.547	.054	.490	.989	1.488
E_o	$i_p r_p$	.0002	.0052	.0102	.0152	.0002	.0042	.0082	.0122	.0002	.0042	.0082	.0122
	$i_p' x_p$	-.001	-.001	-.001	-.001	-.001	-.007	-.013	-.019	-.001	.005	.0110	.0170
	$e_o = e_i + i_p r_p - i_p' x_p$	1.001	1.011	1.021	1.031	1.001	1.021	1.041	1.061	1.001	.997	.993	.989
	$i_p' r_p$	-.0005	-.0005	-.0005	-.0005	-.0005	-.0035	-.0065	-.0095	-.0005	.0025	.0055	.0085
	$i_p x_p$	.0004	.0104	.0204	.0304	.0004	.0084	.0164	.0244	.0004	.0084	.0164	.0244
	$e_o' = e_i' + i_p' r_p + i_p x_p$	-.0001	.020	.040	.060	-.0001	.010	.020	.030	-.0001	.022	.044	.066
	$E_o = \sqrt{e_o^2 + e_o'^2}$	1.001	1.011	1.022	1.033	1.001	1.021	1.041	1.062	1.001	.997	.994	.991
Power	$P = ei$	0	.5	1	1.5	0	.4	.8	1.2	0	.4	.8	1.2
	$e_o i_p$	.02	.5263	1.043	1.570	.02	.429	.854	1.295	.02	.419	.815	1.207
	$e_o' i_p'$	0	-.001	-.002	-.003	0	-.003	-.013	-.028	0	.0045	.024	.056
	$P_o = e_o i_p + e_o' i_p'$	.02	.525	1.041	1.567	.02	.426	.841	1.267	.02	.424	.839	1.263
	$E_o I_p$	.054	.529	1.045	1.573	.054	.559	1.089	1.643	.054	.488	.983	1.475
	$PF_o = P_o / E_o I_p$	.37	.992	.996	.999	.37	.762	.772	.772	.37	.869	.854	.856
	$Eff = P / P_o$	0	.952	.960	.958	0	.939	.951	.947	0	.944	.954	.950
	$Reg. = \frac{E_o - e}{e}$	.001	.011	.022	.033	.001	.021	.041	.062	.001	-.003	-.006	-.009

CALCULATION OF THE TRANSFORMER

Effect of the Variation of the Resistance of the Transformer Upon its Operating Characteristics.

	PF	1	1	1	1	.8	.8 lag	.8	.8	.8	.8 lead	.8	.8
	e	1	1	1	1	1	1	1	1	1	1	1	1
Consts.	$r_p = r_s$	.01	.05	.15	.25	.01	.05	.15	.25	.01	.05	.15	.25
	$x_p = x_s$		.02				.02				.02		
	g		.02				.02				.02		
	b		-.05				-.05				-.05		
	I	1	1	1	1	1	1	1	1	1	1	1	1
I_i	i	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	i'	0	0	0	0	-.6	-.6	-.6	-.6	.6	.6	.6	.6
E_i	ir_s	.01	.05	.15	.25	.008	.04	.12	.20	.008	.04	.12	.20
	$i'x_s$	0	0	0	0	-.012	-.012	-.012	-.012	.012	.012	.012	.012
	$e_i = e + ir_s - i'x_s$	1.01	1.05	1.15	1.25	1.02	1.052	1.132	1.212	.996	1.028	1.108	1.188
	$i'r_s$	0	0	0	0	-.006	-.03	-.09	-.15	.006	.03	.09	.15
	ix_s	.02	.02	.02	.02	.016	.016	.016	.016	.016	.016	.016	.016
	$e_i' = i'r_s + ix_s$	.02	.02	.02	.02	.010	-.014	-.074	-.134	.022	.046	.106	.166
I_{oo}	$e_i g$	.0202	.0210	.0230	.0250	.0204	.0210	.0226	.0242	.0199	.0206	.0222	.0238
	$e_i' b$	-.001	-.001	-.001	-.001	-.0005	.0007	.0037	.0067	-.0011	-.0023	-.0053	-.0083
	$i_{oo} = e_i g - e_i' b$	.0212	.022	.024	.026	.0209	.0203	.0189	.0175	.0210	.0229	.0275	.0321
	$e_i' g$	.0004	.0004	.0004	.0004	.0002	-.0003	-.0015	-.0027	.0004	.0009	.0021	.0033
	$e_i b$	-.0505	-.0525	-.0575	-.0625	-.0510	-.0526	-.0566	-.0606	-.0498	-.0514	-.0554	-.0594
	$i_{oo}' = e_i' g + e_i b$	-.0501	-.0521	-.0571	-.0621	-.0508	-.0529	-.0581	-.0633	-.0494	-.0505	-.0533	-.0561
I_p	$i_p = i + i_{oo}$	1.021	1.022	1.024	1.026	.8209	.8203	.8189	.8175	.8210	.8229	.8275	.8321
	$i_p' = i' + i_{oo}'$	-.050	-.0525	-.0575	-.0625	-.6508	-.6529	-.6581	-.6633	.5506	.5495	.5467	.5439
	$I_p = \sqrt{i_p^2 + i_p'^2}$	1.022	1.023	1.026	1.028	1.048	1.050	1.051	1.053	.989	.989	.990	.992
E_o	$i_p r_p$	.0102	.0511	.1535	.2565	.0082	.0410	.1230	.2044	.0082	.0411	.1243	.2080
	$i_p' x_p$	-.0010	-.0010	-.0011	-.0012	-.0130	-.0130	-.0131	-.0132	.0111	.0110	.0109	.0109
	$e_o = e_i + i_p r_p - i_p' x_p$	1.021	1.102	1.305	1.508	1.041	1.106	1.268	1.430	.993	1.038	1.221	1.385
	$i_p' r_p$	-.0005	-.0026	-.0086	-.0156	-.0065	-.0326	-.0986	-.1655	.0055	.0275	.0820	.1360
	$i_p x_p$	.0204	.0204	.0205	.0205	.0164	.0164	.0164	.0163	.0164	.0165	.0165	.0166
	$e_o' = e_i' + i_p' r_p + i_p x_p$	.04	.038	.032	.025	.02	-.030	-.156	-.283	.044	.090	.204	.319
	$E_o = \sqrt{e_o^2 + e_o'^2}$	1.022	1.103	1.306	1.508	1.041	1.106	1.277	1.465	.994	1.062	1.238	1.420
Power	$P = ei$	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	$e_o i_p$	1.043	1.127	1.337	1.547	.855	.907	1.039	1.170	.815	.871	1.011	1.153
	$e_o' i_p'$	-.002	-.002	-.002	-.002	-.013	.0196	.1025	.187	.024	.049	.111	.173
	$P_o = e_o i_p + e_o' i_p'$	1.041	1.125	1.335	1.545	.842	.927	1.146	1.357	.839	.920	1.122	1.326
	$E_o I_p$	1.044	1.129	1.34	1.55	1.092	1.162	1.340	1.532	.983	1.050	1.225	1.41
	$PF_o = P_o / E_o I_p$	.995	.995	.995	.997	.76	.797	.857	.885	.853	.876	.908	.94
	$Eff = P / P_o$	.96	.89	.75	.647	.95	.863	.700	.592	.953	.870	.720	.603
	$Reg. = \frac{E_o - e}{e}$	.022	.103	.306	.508	.041	.106	.277	.465	-.006	.062	.238	.420

CALCULATION OF THE TRANSFORMER

Effect of the Variation of the Reactance of the Transtormer Upon its Operating Characteristics.

	PF	1	1	1	1	.9	.8 lag	.8	.8	.8	.8 lead	.8	.8
	e	1	1	1	1	1	1	1	1	1	1	1	1
Consts.	$r_p = r_s$		.01				.01				.01		
	$x_p = x_s$	.02	.05	.15	.25	.02	.05	.15	.25	.02	.05	.15	.25
	g		.02				.02				.02		
	b		-.05				-.05				-.05		
	I	1	1	1	1	1	1	1	1	1	1	1	1
I	i	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	i'	0	0	0	0	-.6	-.6	-.6	-.6	.6	.6	.6	.6
	ir_s	.01	.01	.01	.01	.008	.008	.008	.008	.008	.008	.008	.008
	$i'x_s$	0	0	0	0	-.012	-.030	-.090	-.150	.012	.030	.090	.150
E_i	$e_i = e + ir_s - i'x_s$	1.01	1.01	1.01	1.01	1.02	1.038	1.098	1.158	.996	.978	.918	.858
	$i'r_s$	0	0	0	0	-.006	-.006	-.006	-.006	.006	.006	.006	.006
	ix_s	.02	.05	.15	.25	.016	.040	.120	.200	.016	.040	.120	.200
	$e_i' = i'r_s + ix_s$	.02	.05	.15	.25	.010	.034	.114	.194	.022	.046	.126	.206
	$e_i g$	.0202	.0202	.0202	.0202	.0204	.0208	.0220	.0232	.0199	.0196	.0184	.0172
	$e_i' b$	-.0010	-.0025	-.0075	-.0125	-.0005	-.0017	-.0057	-.0097	-.0011	-.0023	-.0063	-.0103
I_{oo}	$i_{oo} = e_i g - e_i' b$	.0212	.0227	.0277	.0327	.0209	.0225	.0277	.0329	.0210	.0219	.0247	.0275
	$e_i' g$	.0004	.001	.003	.005	.0003	.0008	.0023	.0039	.0004	.0009	.0025	.0041
	$e_i b$	-.0505	-.0505	-.0505	-.0505	-.0510	-.0519	-.0549	-.0579	-.0498	-.0489	-.0459	-.0414
	$i_{oo}' = e_i' g + e_i b$	-.0501	-.0495	-.0475	-.0455	-.0507	-.0511	-.0526	-.0540	-.0494	-.0480	-.0434	-.0373
	$i_p = i + i_{oo}$	1.0212	1.0227	1.0277	1.0327	.8209	.8235	.8277	.8329	.8210	.8219	.8241	.8275
I_p	$i_p' = i' + i_{oo}'$	-.0501	-.0495	-.0475	-.0455	-.6501	-.6511	-.6526	-.6540	.5506	.5520	.5566	.5627
	$I_p = \sqrt{i_p^2 + i_p'^2}$	1.022	1.024	1.029	1.034	1.049	1.050	1.053	1.060	.988	.990	.994	1.000
	$i_p r_p$	.0102	.0102	.0103	.0103	.0082	.0082	.0083	.0083	.0082	.0082	.0082	.0083
	$i_p' x_p$	-.0010	-.0025	-.0071	-.0114	-.0130	-.0326	-.0979	-.1635	.0110	.0276	.0835	.1407
E_o	$e_o = e_i + i_p r_p - i_p' x_p$	1.021	1.023	1.027	1.032	1.041	1.079	1.204	1.328	.993	.959	.843	.726
	$i_p' r_p$	-.0005	-.0005	-.0005	-.0005	-.0065	-.0065	-.0065	-.0065	.0055	.0055	.0056	.0056
	$i_p x_p$	.0204	.0511	.1541	.2582	.0164	.0412	.1241	.2082	.0164	.0411	.1237	.2069
	$e_o' = e_i' + i_p' r_p + i_p x_p$	.04	.101	.304	.508	.02	.069	.232	.396	.044	.093	.255	.419
	$E_o = \sqrt{e_o^2 + e_o'^2}$	1.022	1.028	1.072	1.151	1.041	1.081	1.228	1.387	.994	.963	.880	.838
	$P = ei$	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
Power	$e_o i_p$	1.043	1.046	1.056	1.066	.855	.888	.996	1.106	.815	.788	.695	.601
	$e_o' i_p'$	-.002	-.005	-.014	-.023	-.013	-.045	-.151	-.260	.024	.051	.142	.236
	$P_o = e_o i_p + e_o' i_p'$	1.041	1.041	1.042	1.043	.842	.843	.845	.846	.839	.839	.837	.837
	$E_o I_p$	1.045	1.053	1.103	1.191	1.092	1.135	1.293	1.470	.982	.953	.875	.838
	$PF_o = P_o / E_o I_p$	.994	.991	.945	.871	.772	.750	.654	.575	.853	.88	.955	1.000
	$Eff = P / P_o$	.960	.960	.960	.959	.950	.948	.946	.945	.953	.953	.954	.955
	$Reg. = \frac{E_o - e}{e}$	.022	.028	.072	.151	.041	.081	.228	.387	-.006	-.037	-.120	-.162

CALCULATION OF THE TRANSFORMER

Effect of the Variation of the Coreloss of the Transformer Upon its Operating Characteristics.

	P.F.	1	1	1	1	.8	.8 lag	.8	.8	.8	.8 lead	.8	.8
	e	1	1	1	1	1	1	1	1	1	1	1	1
Constants	$r_p = r_s$		.01				.01				.01		
	$x_p = x_s$		.02				.02				.02		
	g	.02	.10	.30	.50	.02	.10	.30	.50	.02	.10	.30	.50
	b		.05				-.05				-.05		
	I	1	1	1	1	1	1	1	1	1	1	1	1
I	i	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	i'	0	0	0	0	-.6	-.6	-.6	-.6	.6	.6	.6	.6
	ir_s		.01				.008				.008		
	$i'x_s$		0				-.012				.012		
E_i	$e_i = e + ir_s - i'x_s$		1.01				1.02				.996		
	$i'r_s$		0				-.006				.006		
	ix_s		.02				.016				.016		
	$e_i' = i'r_s + ix_s$		.02				.010				.022		
	$e_i g$	.0202	.101	.303	.505	.0204	.102	.306	.510	.0199	.0996	.2988	.498
	$e_i' b$	-.001	-.001	-.001	-.001	-.0005	-.0005	-.0005	-.0005	-.001	-.001	-.001	-.001
I_{oo}	$i_{oo} = e_i g - e_i' b$	0212	.102	.304	.506	.0209	.1025	.3065	.5105	.021	.100	.300	.499
	$e_i' g$	.0004	.002	.006	.010	0003	.001	.003	.005	.0004	.0022	.0066	.011
	$e_i b$	-.050	-.0505	-.0505	-.0505	-.051	-.051	-.051	-.051	-.0498	-.0498	-.0498	-.0498
	$i_{oo}' = e_i' g + e_i b$	-.05	-.0485	-.0445	-.0403	-.0507	-.050	-.048	-.046	-.0494	.0476	-.043	-.033
	$i_p = i + i_{oo}$	1.0	1.102	1.304	1.506	.8209	.9025	1.106	1.310	.821	.900	1.100	1.300
I_p	$i_p' = i' + i_{oo}'$	-.05	-.0495	-.0495	-.0403	-.6511	-.650	-.648	-.646	.551	.552	.557	.561
	$I_p = \sqrt{i_p^2 + i_p'^2}$	1.02	1.103	1.305	1.507	1.099	1.112	1.282	1.460	.988	1.055	1.233	1.415
	$i_p r_p$	.0102	.0110	.0130	.0151	.0082	.0091	.0111	.0131	.008	.009	.011	.013
	$i_p' x_p$	-.001	-.001	-.0009	-.0008	-.013	-.013	-.013	.013	.0110	.0110	.0111	.0112
	$e_o = e_i + i_p r_p - i_p' x_p$	1.021	1.022	1.024	1.026	1.041	1.042	1.044	1.046	.993	.994	.996	.998
E_o	$i_p' r_p$	-.0005	-.0005	-.0005	-.0004	-.0065	-.0065	-.0065	-.0065	.0055	.0055	.0056	.0056
	$i_p x_p$	.0204	.0220	.0261	.0301	.0164	.0181	.0222	.0263	.0164	.018	.022	.026
	$e_o' = e_i' + i_p' r_p + i_p x_p$	.040	.0415	.0455	.0495	.020	.022	.026	.030	.044	.045	.050	.055
	$E_o = \sqrt{e_o^2 + e_o'^2}$	1.022	1.023	1.025	1.027	1.041	1.042	1.044	1.047	.994	.995	.997	.999
Power	$P = ei$	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	$e_o i_p$	1.043	1.126	1.335	1.545	.855	.941	1.155	1.370	.815	.894	1.095	1.297
	$e_o' i_p'$	-.002	-.002	-.002	-.002	-.013	-.014	-.017	-.019	.024	.024	.028	.031
	$P_o = e_o i_p + e_o' i_p'$	1.041	1.124	1.333	1.543	.842	.927	1.138	1.351	.839	.918	1.123	1.328
	$E_o I_p$	1.045	1.128	1.338	1.548	1.092	1.160	1.338	1.53	.982	1.050	1.230	1.413
	$PF_o = P_o / E_o I_p$	.996	.996	.996	997	.772	.799	.850	883	.853	874	913	.94
	$Eff = P / P_o$	.960	.890	.750	.648	.950	.863	.703	.592	.953	.872	.712	.604
	$Reg = \frac{E_o - e}{e}$	.022	.023	.025	.027	.041	.042	.044	.047	-.006	-.005	-.003	-.001

CALCULATION OF THE TRANSFORMER

Effect of the Variation of the Magnetizing Current of the Transformer Upon its Operating Characteristics.

	P.F. =	1	1	1	1	.8	.8 lag	.8	.8	.8	.8 lead	.8	.8
	e =	1	1	1	1	1	1	1	1	1	1	1	1
Constants.	$r_p = r_s$ =		.01				.01				.01		
	$x_p = x_s$ =		.02				.02				.02		
	g =		.02				.02				.02		
	b =	-.05	-.10	-.30	-.50	-.05	-.10	-.30	-.50	-.05	-.10	-.30	-.50
	I	1	1	1	1	1	1	1	1	1	1	1	1
I	i	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	i'	0	0	0	0	-.6	-.6	-.6	-.6	.6	.6	.6	.6
	$i r_s$		.01				.008				.008		
	$i' x_s$		0				-.012				.012		
E_i	$e_i = e + i r_s - i' x_s$ =		1.01				1.02				.996		
	$i' r_s$		0				-.006				.006		
	$i x_s$		.02				.016				.016		
	$e_i' = i' r_s + i x_s$ =		.02				.010				.022		
	$e_i g$	.0202	.0202	.0202	.0202	.0204	.0204	.0204	.0204	.0199	.0199	.0199	.0199
	$e_i' b$	-.001	-.002	-.006	-.01	-.0005	-.001	-.003	-.005	-.001	-.0022	-.0066	-.0110
I_{oo}	$i_{oo} = e_i g - e_i' b$ =	.0212	.0222	.0262	.0302	.0209	.0214	.0234	.0254	.0210	.0221	.0265	.0309
	$e_i' g$	.0004	.0004	.0004	.0004	.0002	.0002	.0002	.0002	.0004	.0004	.0004	.0004
	$e_i b$	-.0505	-.101	-.303	-.505	-.051	-.102	-.306	-.510	-.0498	-.0996	-.2988	-.4980
	$i'_{oo} = e_i' g + e_i b$ =	-.0501	-.1006	-.3026	-.5046	-.0508	-.1018	-.3058	-.5098	-.0494	-.0992	-.2984	-.4976
	$i_p = i + i_{oo}$ =	1.0212	1.0222	1.0262	1.0302	.8209	.8214	.8234	.8254	.8210	.8221	.8265	.8309
I_p	$i'_p = i' + i'_{oo}$ =	-.0505	-.1006	-.3026	-.5046	.6507	-.7018	-.9058	-1.1098	.551	.5008	.3016	.1024
	$I_p = \sqrt{i_p^2 + i_p'^2}$ =	1.022	1.027	1.071	1.146	1.049	1.080	1.223	1.383	.988	.963	.881	.837
	$i_p r_p$	.0102	.0102	.0103	.0103	.0082	.0082	.0082	.0083	.0080	.0082	.0083	.0083
	$i'_p x_p$	-.001	-.002	-.006	-.0101	-.013	-.0140	-.0181	-.0222	.0110	.010	.006	.002
	$e_o = e_i + i_p r_p - i'_p x_p$ =	1.021	1.0222	1.0263	1.0304	1.041	1.0422	1.0463	1.0505	.993	.9942	.9982	1.0022
E_o	$i'_p r_p$	-.0005	-.0010	-.0030	-.0050	-.0065	-.0070	-.0091	-.0111	.0055	.0050	.0030	.0010
	$i_p x_p$	.0204	.0204	.0205	.0206	.0164	.0164	.0165	.0165	.0164	.0164	.0165	.0166
	$e'_o = e_i' + i'_p r_p + i_p x_p$ =	.040	.0394	.0375	.0356	.020	.0194	.0174	.0154	.044	.0434	.0415	.0396
	$E_o = \sqrt{e_o^2 + e_o'^2}$ =	1.022	1.023	1.027	1.031	1.041	1.043	1.047	1.051	.994	.995	.999	1.003
Power	P = ei =	1	1	1	1	.8	.8	.8	.8	.8	.8	.8	.8
	$e_o i_p$	1.043	1.044	1.052	1.061	.855	.856	.862	.868	.815	.817	.826	.833
	$e'_o i'_p$	-.002	-.004	-.011	-.018	-.013	-.014	-.016	-.017	.024	.022	.013	.004
	$P_o = e_o i_p + e'_o i'_p$ =	1.041	1.040	1.041	1.043	.842	.842	.846	.851	.839	.839	.839	.837
	$E_o I_p$	1.045	1.051	1.100	1.180	1.092	1.126	1.281	1.455	.982	.958	.880	.840
	$PF_o = P_o / E_o I_p$ =	.996	.990	.945	.886	.772	.75	.66	.585	.853	.878	.953	.996
	$Eff. = P / P_o$ =	.960	.960	.960	.959	.950	.950	.946	.940	.954	.954	.954	.956
	$Reg. = \frac{E_o - e}{e}$ =	.0222	.023	.027	.031	.041	.043	.047	.051	-.006	-.005	-.001	.003

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